Example

matrix	A_1	A_2	A_3	A_4	A_5	A_6
dimension	4x2	2x3	3x1	1x2	2x2	2x3

Table m

M	A_1	A_2	A_3	A_4	A_5	A_6
A_1	0	24	14	22	26	36
A_2		0	6	10	14	22
A_3			0	6	10	19
A_4				0	4	10
A_5					0	12
A_6						0

2	3	4	5	6	
1	1	3	3	3	1
	2	3	3	3	2
		3	3	3	3
			4	5	4
				5	5

Computeng the Optimal Costs :-

$l=2$

m[1,2], m[2,3], m[3,4], m[4,5], m[5,6]

$l=3$

m[1,3], m[2,4], m[3,5], m[4,6]

Compute each one and store the min value in the table m

$l=4$

m[1,4], m[2,5], m[3,6]

$l=5$

m[1,5], m[2,6]

$l=6$

m[1,6]

$m[1,2] = 4 \times 2 \times 3 = 24$

$m[2,3] = 2 \times 3 \times 1 = 6$

$m[3,4] = 3 \times 1 \times 2 = 6$

$m[4,5] = 1 \times 2 \times 2 = 4$

$m[5,6] = 2 \times 2 \times 3 = 12$

$$m[1,3] = \min \begin{cases} \overset{0}{m[1,1]} + \overset{6}{m[2,3]} + P_0 P_1 P_3 = 0 + 6 + 4 \times 2 \times 1 = 14 \quad k=1 \\ m[1,2] + m[3,3] + P_0 P_2 P_3 = 24 + 0 + 4 \times 3 \times 1 = 36 \end{cases}$$

$$m[2,4] = \min \begin{cases} m[2,2] + m[3,4] + P_1 P_2 P_4 = 0 + 6 + \overset{2\times3\times2}{12} = 18 \\ m[2,3] + m[4,4] + P_1 P_3 P_4 = 6 + 0 + \underset{2\times1\times2}{4} = 10 \quad k=3 \end{cases}$$

$$m[3,5] = \min \begin{cases} m[3,3] + m[4,5] + P_2 P_3 \overset{3\times1\times2}{P_5} = 0 + 4 + 6 = 10 \quad k=3 \\ m[3,4] + m[5,5] + P_2 P_4 P_5 = 6 + 0 + 12 = 18 \end{cases}$$

$$m[4,6] = \min \begin{cases} m[4,4] + m[5,6] + P_3 P_4 P_6 = 0 + 12 + 6 = 18 \\ m[4,5] + m[6,6] + P_3 P_5 P_6 = 4 + 0 + 6 = 10 \quad k=5 \end{cases}$$

$$m[1,4] = \min \begin{cases} min[1,1] + m[2,4] + P_0 \times P_1 \times P_4 = 0 + 10 + \overset{4\times2\times2}{16} = 26 \\ min[1,2] + m[3,4] + P_0 \times P_2 \times P_4 = 24 + 6 + 24 = 54 \\ min[1,3] + m[4,4] + P_0 \times P_3 \times P_4 = 14 + 0 + 8 = 22 \quad k=3 \end{cases}$$

$$m[2,5] = \min \begin{cases} m[2,2] + m[3,5] + P_1 P_2 P_5 = 0 + 10 + \overset{2\times3\times2}{12} = 22 \\ m[2,3] + m[4,5] + P_1 P_3 P_5 = 6 + 4 + 4 = 14 \quad k=3 \\ m[2,4] + m[5,5] + P_1 P_4 P_5 = 10 + 0 + 8 = 18 \end{cases}$$

$$m[3,6] = \min \begin{cases} m[3,3] + m[4,6] + P_2 P_3 P_6 = 0 + 10 + \overset{3\times1\times3}{9} = 19 \quad k=3 \\ m[3,4] + m[5,6] + P_2 P_4 P_6 = 6 + 12 + \overset{3\times2\times3}{18} = 36 \\ m[3,5] + m[6,6] + P_2 P_5 P_6 = 10 + 0 + \overset{3\times2\times3}{18} = 28 \end{cases}$$

$$m[1,5] = \min \begin{cases} m[1,1] + m[2,5] + P_0 P_1 P_5 = 0+14+16 = 30 \\ m[1,2] + m[3,5] + P_0 P_2 P_5 = 24+10+24 = 58 \\ m[1,3] + m[4,5] + P_0 P_3 P_5 = 14+4+8 = 26 \quad k=3 \\ m[1,4] + m[5,5] + P_0 P_4 P_5 = 22+0+16 = 38 \end{cases}$$

$$m[2,6] = \min \begin{cases} m[2,2] + m[3,6] + P_1 P_2 P_6 = 0+19+18 = 37 \\ m[2,3] + m[4,6] + P_1 P_3 P_6 = 6+10+6 = 22 \quad k=3 \\ m[2,4] + m[5,6] + P_1 P_4 P_6 = 10+12+12 = 34 \\ m[2,5] + m[6,6] + P_1 P_5 P_6 = 14+0+12 = 26 \end{cases}$$

* * *

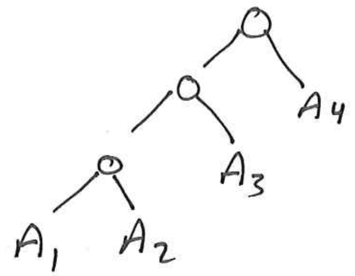
$$m[1,6] = \min \begin{cases} m[1,1] + m[2,6] + P_0 P_1 P_6 = 0+22+24 = 46 \\ m[1,2] + m[3,6] + P_0 P_2 P_6 = 24+19+36 = 79 \\ m[1,3] + m[4,6] + P_0 P_3 P_6 = 14+10+12 = 36 \quad k=3 \\ m[1,4] + m[5,6] + P_0 P_4 P_6 = 22+12+24 = 58 \\ m[1,5] + m[6,6] + P_0 P_5 P_6 = 26+0+24 = 50 \end{cases}$$

Example :- Matrix Chain Multiplication Problem

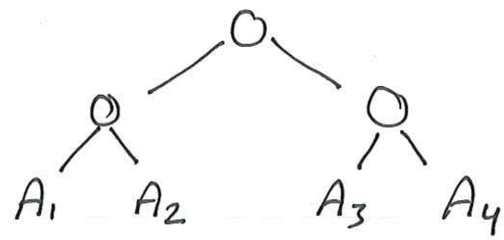
A_1	A_2	A_3	A_4
5×4	4×6	6×2	2×7

How to multiply?

$$((A_1 \cdot A_2) \cdot A_3) \cdot A_4)$$



$$(A_1 \cdot A_2) \cdot (A_3 \cdot A_4)$$



How to find the minimum number of multiplications to multiply the 4 matrecies?

* Using Dynamic Programing

m	1	2	3	4
1	0	120	88	158
2		0	48	104
3			0	84
4				0

$L = 1 \Rightarrow$ All zeros

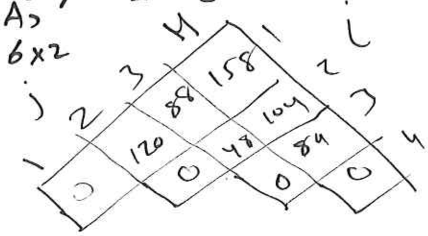
✓ A_1, A_2, A_3, A_4

$L = 2$

$m[1,2]$, $m[2,3]$, $m[3,4]$

$A_1 \cdot A_2$ $\underset{4 \times 6}{A_2}$ $\underset{6 \times 2}{A_3}$

$5 \times 4 \quad 4 \times 6$



S	1	2	3	4
1		1	1	3
2		~~2~~	2	3
3				3
4				

$L = 3$

$m[1,3]$,

$(\underset{5 \times 4}{A_1} \cdot (A_2 \quad A_3))$ $((\underset{5 \times 4}{A_1} \quad \underset{4 \times 6}{A_2}) \cdot \underset{6 \times 2}{A_3})$

$m[1,1] + m[2,3] + 5 \times 4 \times 2$	$m[1,2] + m[3,3] + 5 \times 6 \times 2$
$A_1 \quad A_2 \quad A_3 \quad = 48 + 40$	$120 + 0 + 120$
$5 \times 4 \quad 4 \times 6 \quad 6 \times 2 \quad = 88$	80
$5 \times 4 \quad 4 \times 2$	✓ 88
$5 \times 4 \times 2$	

$m[2,4] = 104$

$104 \rightarrow k = 3$

$A_2 \, (A_3 \; A_4)$ $\qquad$ $(A_2 \; A_3) \, A_4$

$4 \times 6 \quad 6 \times 2 \quad 2 \times 7$

$m[2,2] + m[3,4] + 4 \times 6 \times 7$ $\qquad$ $m[2,3] + m[4,4] + 4 \times 2 \times 7$

$0 + 84 + 168 = 252$ $\qquad$ $48 + 0 + 56 = 104$

$L = 3$

$A_1 \quad A_2 \quad A_3 \quad A_4$

$5 \times 4 \quad 4 \times 6 \quad 6 \times 2 \quad 2 \times 7$

$$m[1,4] = \min \begin{cases} m[1,1] + m[2,4] + 5 \times 4 \times 7, \quad m[1,2] + m[3,4] + 5 \times 6 \times 7 \\ 0 + 104 + 140 \qquad\qquad 120 + 84 + 210 \\ m[1,3] + m[4,4] + 5 \times 2 \times 7 \\ 88 + 0 + 70 = 158 \end{cases}$$

$k = 3$

$5 \Rightarrow 3$

Formula

$$m[i,j] = \min \left\{ m[i,k] + m[k+1,j] + P_{i-1} * P_k * P_j \right.$$

$A_1 \qquad A_2 \qquad A_3 \qquad A_4$

$5 \times 4 \qquad 4 \times 6 \qquad 6 \times 2 \qquad 2 \times 7$

$P_0 \qquad P_1 \qquad P_2 \qquad P_3 \qquad P_4$

time complexity

because of zeros

$$\frac{n(n-1)}{2} = n^2 \times n \Rightarrow O(n^3)$$

we calculate only half of the table

for each value in the table, we find all possible values, n times

0			
	0		
		0	
			0

m

	0	1	2	3	4
0	0	0	0	0	0
1	0	0			
2	0		0		
3	0			0	
4	0				0

s

	0	1	2	3	4
0	0	0	0	0	0
1	0	0			
2	0		0		
3	0			0	
4	0				0

i	j
1 +	2
2 +	3
3	4

i	j
1	3
2	4

i	j
1	4

differency

d = 1 d = 2 d = 3

calculate diagonally

so the most outer loop becomes: for (d = 1 ; d < n-1 ; d++)

n = 5

how to find j ??

d + i ✓

d < n-1 ?
why n-1

the first time the row will reach 3 then 2 then 1. 3 is 4-1

```
{
  P[5]={5,4,6,2,7}   -> dimensions.
  m[5][5] = {0};
  s[5][5]={0};
  int j, min, q;

  for (int d=1; d<n-1 ;d++)
     for (i=1; i<n-d ; i++)
     {
        j = i+d
        min = ∞ ;
        for (int k=1; k<=j-1 ; k++)
        {
           q= m[i][k] + m[k+1][j]+ P[i-1]* P[k]* P[j];
           if(q<min)
           {
              min = q;
              s[i][j] = k;
           }
        }
        m[i][j] = min;
     }
   }
}
```

$$\underset{5\times 4}{A_1} \times \underset{4\times 6}{A_2} \times \underset{6\times 2}{A_3} \times \underset{2\times 7}{A_4}$$

$P_0 \quad P_1 \quad P_2 \quad P_3 \quad P_4$